A Study on Computational of Flow Modeling

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Abstract

A finite element method (FEM) analysis is carried out for the determination of confined turbulent flow with a one-equation model used to depict the turbulent viscosity as applied to a smooth straight channel. The effects of such flow in a zone close to a solid boundary have been investigated; as a result of this, a wall element technique based on the use of FEM was developed. Parabolic elements in one direction, normal to the wall, and in two directions, normal and parallel to the wall, have been adopted and tested in a zone close to the solid wall, which replaces the traditional use of the empirical laws. The validity of the technique was examined for developing and fully developed turbulent flow and compared well with other standard techniques.

Introduction

Due to digital growth of technology and the importance of fluid dynamics applications, computational fluid dynamics has interested researchers. It is well known that when a fluid enters a prismoidal duct, the values of the pertinent variables change from some initial profile to a fully developed form, which is thereafter invariant in the downstream direction. Numerous theoretical and experimental works are available on laminar flow [1-2], but in the case of turbulent flow, there are few. Since it has not been possible to obtain exact analytical solutions to such flows, an accurate numerical approach is needed, an effective technique to model the variation of the pertinent variables near the solid boundary, where the variation in velocity and kinetic energy, in particular, is extremely large since the transfer of shear from the boundary into the main domain and the nature of the flow changes rapidly. Consequently, if a conversational finite element were used to model the near wall zone (NWZ), a significant grid refinement would be required.

Several solution techniques have been suggested in order to avoid such excessive refinement [3-5]. A more common approach is to terminate the main domain subject to discretisation at some small distance away from the wall, where the gradients of the independent variables are relatively small, and then use another technique to model the flow behavior in the near wall zone. In previous work, a wall element technique based on FE using parabolic elements, in one direction normal to the solid wall as shown in Figure 1, has been adopted [6] and applied successfully to combination of pressure and Coquette flow. Also, the validity of this technique has been examined for developing turbulent flow in a straight channel with fixed walls, even when the NWZ was extended away from the fixed solid wall. It has been proven that using universal laws is not valid for both developing and fully developed flow, and the general use of 2-D elements up to the wall is not economically viable. Also, the use of the wall element technique using 1-D in one direction normal to the wall has approved to be valid and superior to other techniques when fully developed turbulent flow was considered but not for developing flow. As a result, the technique has been modified in this paper by using parabolic elements in two directions normal and parallel to the solid wall. It tested and compared well with other techniques when developing and fully developed turbulent flow was considered.

Governing Equations

The Navier-Stokes equations associated with steady state incompressible two-dimensional turbulent flow of a Newtonian viscous fluid with no body forces acting are

$$\rho u_j \frac{\partial u_i}{\partial x_j} = - \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\mu_e\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)\right] \quad (1)$$

where i,j= 1,2. u_i, p are the velocities and pressure respectively, ρ is the fluid density, μ_e is the effective viscosity which is given by $\mu_e = \mu + \mu_t$, μ and μ_t are the molecular viscosity and turbulent viscosity. The continuity equation can be written as

$$\frac{\partial u_i}{\partial x_i} = 0 \quad (2)$$

Equations 1 and 2 cannot be solved unless a turbulence closure model can be provided to evaluate the turbulent contribution to μ_e. In the present work, a one-equation model has been adopted so that

$$\mu_t = C_\mu \rho k^{1/2} 1_\mu \quad (3)$$

1_μ is the length scale which is taken as 0.4 times the normal distance from the nearest wall surface, C_μ is a constant. The distribution of the turbulence kinetic energy k [7-8] can be evaluated by the transport equation,

$$\rho u_j \frac{\partial k}{\partial x_j} = \frac{\partial}{\partial x_j}\left[\left(\mu+\frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right]+\mu_t \frac{\partial u_i}{\partial x_j}\left[\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right]-E \quad (4)$$

Where E= $C_D \rho k^{3/2} / 1_\mu$, μ_t / σ_k is the turbulent diffusion coefficient, σ_k is the turbulent prandtl or Schmidt number and C_D is a constant. The turbulence model based on equations 1-2 and 4 are called the one-equation (k-l) model. The above governing equations have been solved using the standard finite element method [9-12], and Galerking weight residual approach is adopted to solve the discretised equations governing the fluid motion. Then quadratical 8-noded elements were used to define the variations in velocities and turbulent kinetic energy, while the pressure was predicted using the mixed interpolation technique. This means linear 4-nodded elements should be used for the pressure. Green's theorem is then used to reduce the order of the equations to unity resulting in a "weak formulation," which in turn resulted in a non-linear equations matrix that is solved by either a coupled or an uncoupled method.

Boundary Conditions

Pressure flow was considered. At the upstream, constant values were imposed for developing turbulent flow, compatible fully developed values were imposed for fully developed turbulent flow and updated until a converged condition is satisfied. No slip condition was imposed on solid boundaries and tractions updated downstream. Tractions are given by

$$\tau_{x_1} = -p + \frac{\mu_e}{\rho}\left(\frac{\partial u_1}{\partial x_1}\right) \qquad x_1\text{- parallel to walls}$$

$$\tau_{x_2} = \frac{\mu_e}{\rho}\left(\frac{\partial u_2}{\partial x_1} + \frac{\partial u_1}{\partial x_2}\right) \qquad x_2\text{- normal to walls}$$

Within the main domain, conventional two-dimensional isoparametric elements are used to discretise the flow domain, and within the NWZ, either the universal laws concept [13] or conventional finite elements (2-D elements up to the wall). In this paper, a wall element technique based on finite elements method has been adopted, using one-dimensional (3-noded elements) normal to the wall as shown in Figure 1, and one-dimensional (3-noded elements) in two directions and normal and parallel to the wall, as shown in Figure 2.

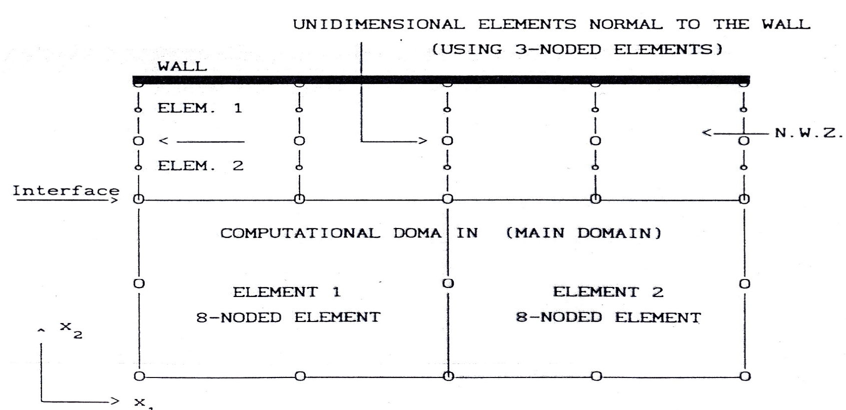

Figure 1. One-dimensional elements in one-direction normal to the wall used in the NWZ

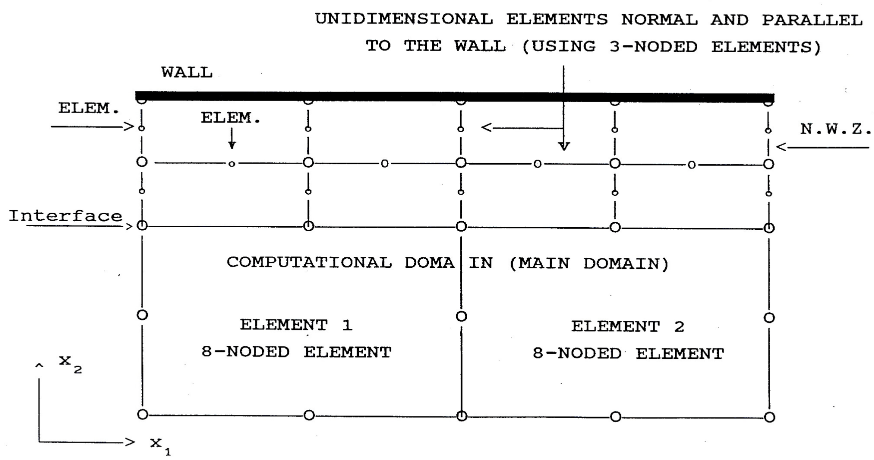

Figure 2. One-dimensional elements in two directions, normal and parallel to the near wall

Results and Discussion

Turbulent flow was considered in a parallel-sided duct with fixed walls of width D, which is taken as 1.0 and length L. Different Reynolds numbers based upon the width of the channel of 1.000, 12.000, and 50.000 were considered.

First Stage: Study of Fully Developed Turbulent Flow

In this stage, fully developed turbulent flow was considered with the adoption of a wall element technique using parabolic elements in two directions, normal and parallel to the solid wall in the NWZ as shown in Figure 2 and compared well with other techniques. Figure 3 clearly shows that convergent fully developed velocity values at downstream obtained by universal profiles have some discrepancy from those obtained from the identical complete mesh (2-D up to the wall) and the advocated technique (1-D in two directions).

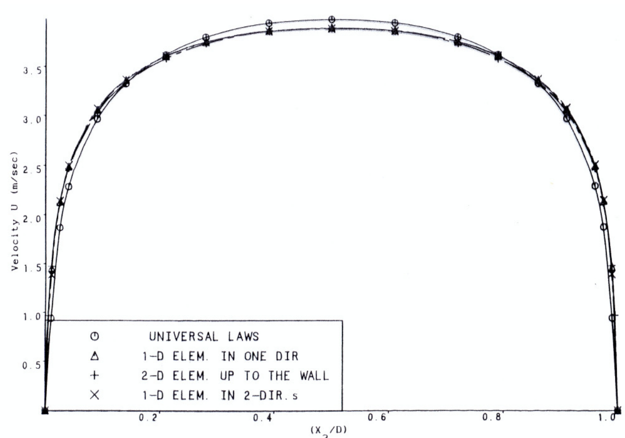

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Figure 3. Fully developed velocity profiles for turbulent flow,
at 8D downstream, Re=12.000

Figures 4-5 refer to the convergent fully developed kinetic energy and viscosity profiles, which clearly s that the results obtained from the complete mesh and advocated technique using 1-D in one direction normal to the wall and the use of 1-D in two directions, normal and parallel to the wall, are identical.

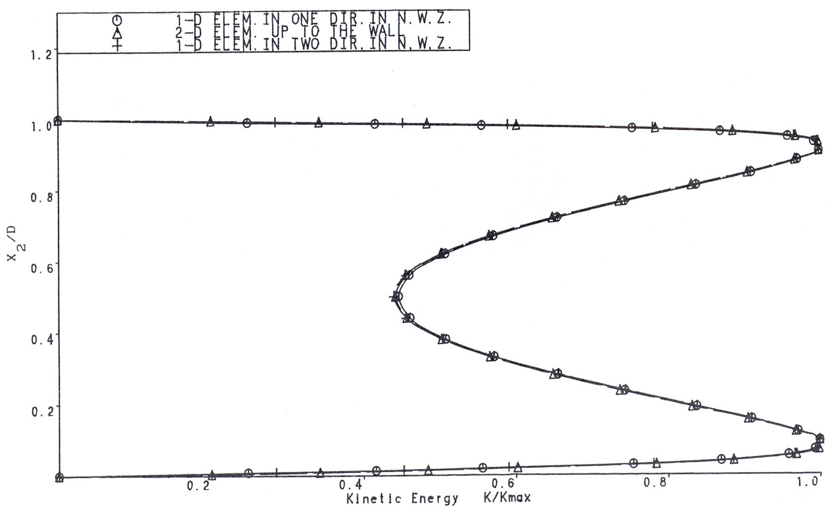

Figure 4. Fully developed kinetic energy profiles for turbulent flow,
at 8D downstream, Re=12.000

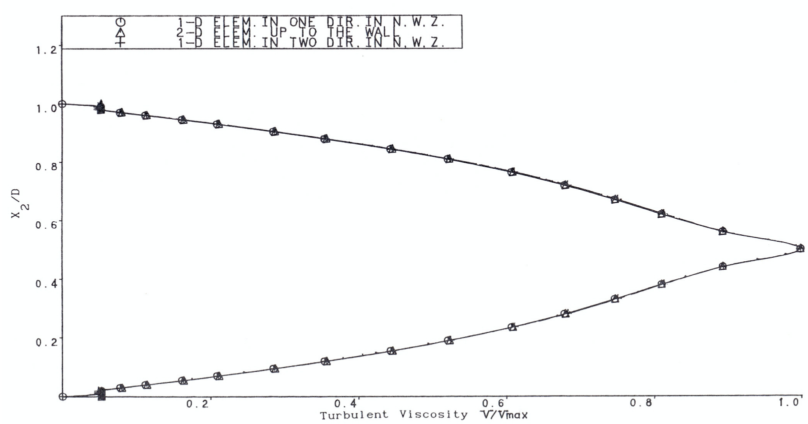

Figure 5. Fully developed viscosity distribution profiles for turbulent flow,
at 1.4D downstream, Re=1.000

Figure 6 represents the longitudinal pressure drops distribution along the near wall nodes and clearly shows that the results obtained from the advocated technique is closer to the correct values than those obtained from universal laws.

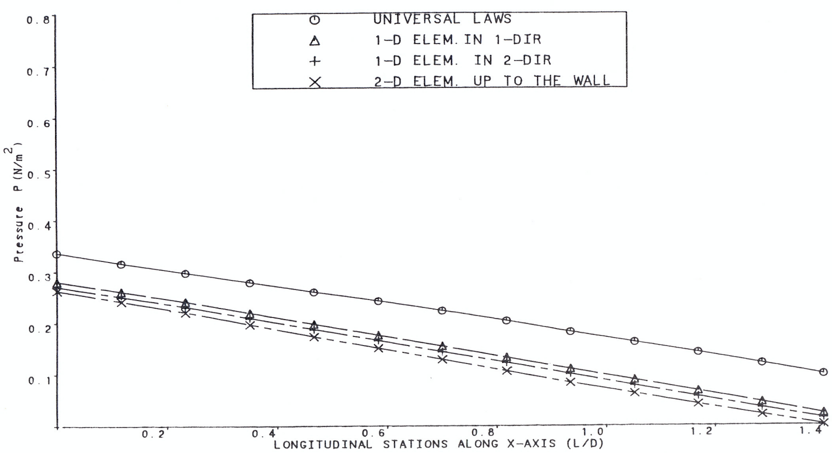

Figure 6. Pressure distribution along the channel, for fully developed flow along the near wall nodes, L=1.4D, Re=1.000

Figure 7 shows excellent agreement between the adopted technique and experimental results [14] for a fully developed velocity profile.

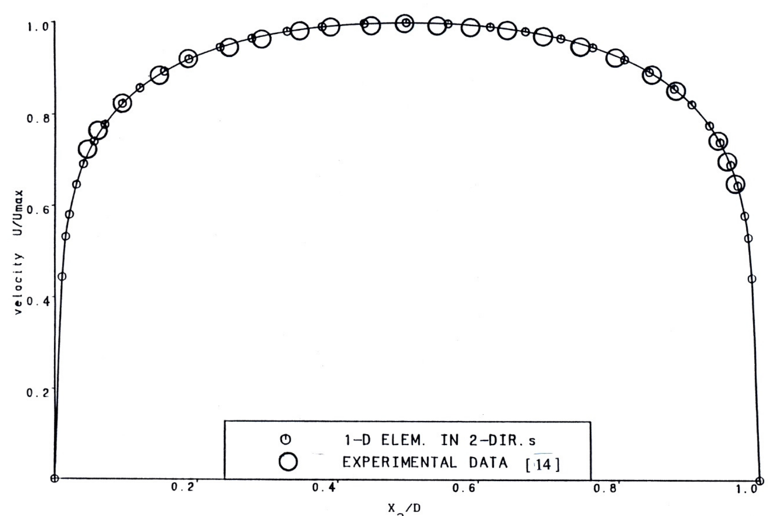

Figure 7. Fully developed velocity profiles for turbulent flow, at 8D downstream, Re=50.000

The conclusion of this stage is that the use of the universal laws technique is not acceptable anymore, the use of 2-D elements up to the wall is not economically viable, and the accuracy of the adopted technique when used in one direction or in two directions is clearly valid for fully developed flow.

Second Stage: Study of Developing Turbulent Flow

Previous work [6] found that the use of the wall element technique using 1-D in one direction normal to the wall is not valid for developing turbulent flow. As a result, the adopted technique has been modified in by using parabolic elements in two directions, normal and parallel to the solid wall , and tested well when compared with other techniques. Convergent velocity profiles are shown in Figure 8, when the velocity distribution was assumed to be 1/7 power law, and the turbulent kinetic energy was assigned as a constant value of 0.005 m^2/sec^2 were imposed at the upstream section. The disparity between the advocated technique and universal laws is now even greater than those for fully developed flow.

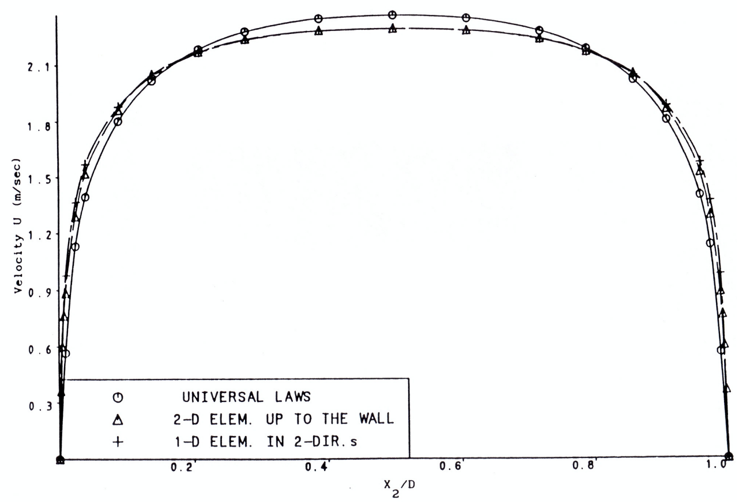

Figure 8. Developing velocity profiles for turbulent flow,
at 10D downstream, Re=12.000

Figure 9 clearly shows that the results obtained from the complete mesh (2-D up to the wall) and the advocated technique (1-D in two directions) are identical and different from those obtained when 1-D in normal direction is used.

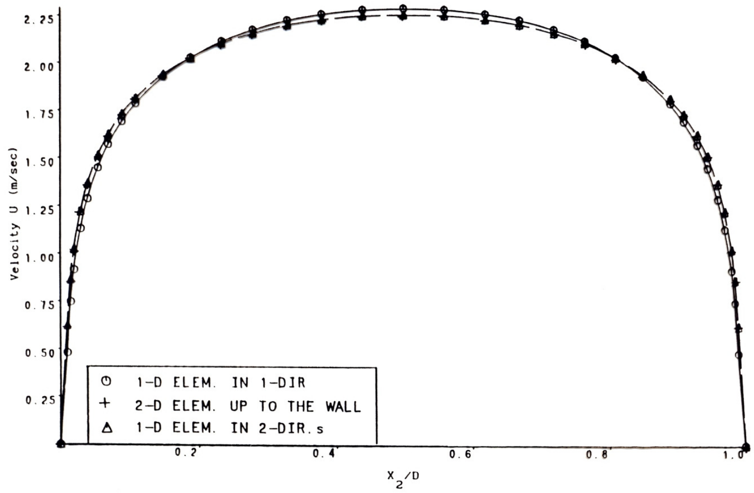

Figure 9. Developing velocity profiles for turbulent flow,
at 10D downstream, Re=12.000

Figures 10-11 represent the downstream kinetic energy turbulent profiles and the turbulent viscosity profiles, respectively. In summary, these show that when 1-D elements in two directions are used, these are superior to those obtained when 1-D elements in one direction only were used. This was known, conceptually, but the variation has now been demonstrated. These results prove that the use of a one-dimensional element in two directions is a valid technique for developing flow, and one can avoid both the mapping of 2-D elements up to the wall and the use of 1-D elements in one direction.

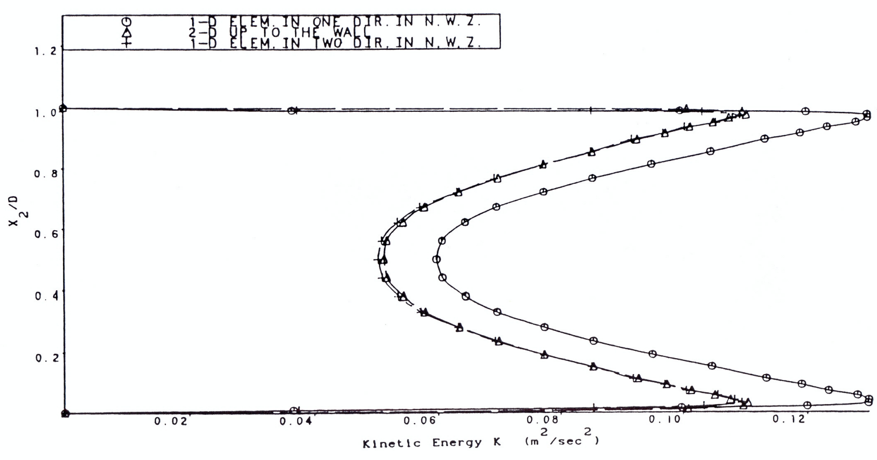

Figure 10. Developing kinetic energy profiles for turbulent flow,
at 10D downstream, Re=12.000

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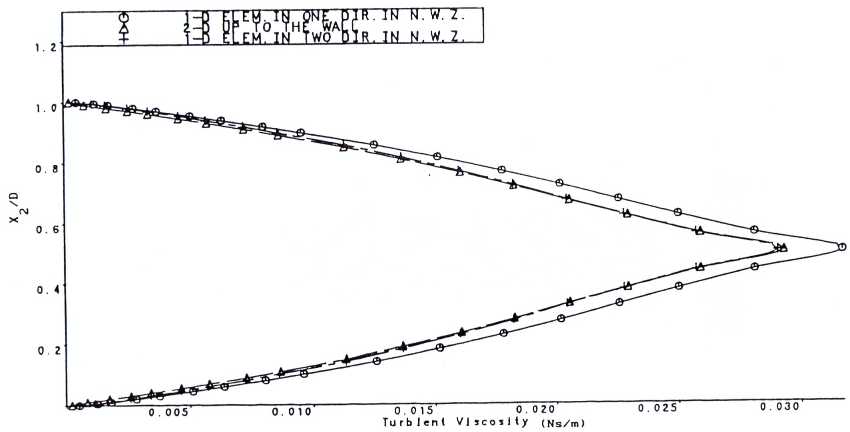

Figure 11. Viscosity profiles for developing turbulent flow, at 10D downstream, Re=12.000

Figure 12 shows the longitudinal shear stress distribution for developing turbulent flow along the near wall nodes and clearly proved that the use of 1-D in two directions is the suitable technique for developing turbulent flow.

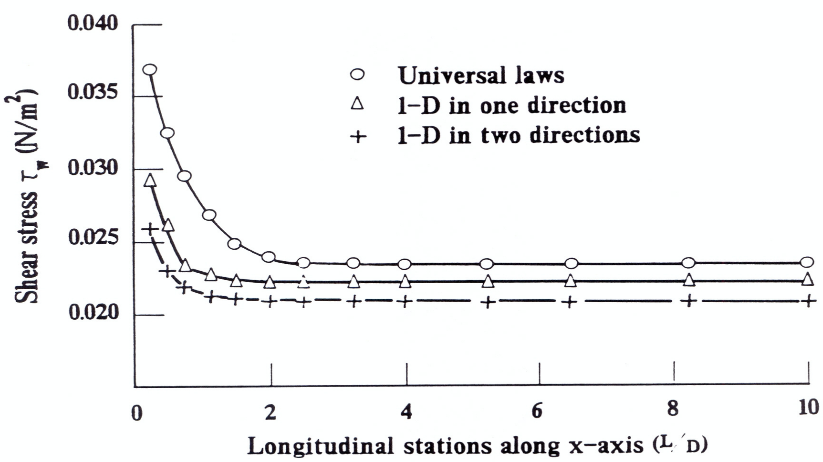

Figure 12. Longitudinal shear stress distribution for developing turbulent flow along the near wall nodes, L=10D, Re=12.000

The conclusion of this stage is that the use of the universal laws technique is no longer acceptable for both developing and fully developed flow. Also, the use of 2-D elements up to the wall is not economically viable, and the adopted technique of using 1-D in two directions can be used with confidence and replaces other techniques when developing turbulent flow is considered.

Conclusions

Using empirical universal laws is no longer acceptable for both developing and fully developed flow, since these laws are only applicable for certain unidimensional flow regimes. Also, the use of 2-D elements up to the wall is not economically viable, since it needs an excessive refinement which is very costly in computer time and memory size. Therefore, an alternative wall element technique has been adopted based on the use of the FEM. The accuracy of this technique when used in one direction normal to the wall has been applied successfully and approved to be superior to other techniques for fully developed turbulent flow. However, this is not the case for developing turbulent flow since the assumption of unidirectional flow is unacceptable. Therefore, this technique has been modified by using 1-D elements in two directions, normal and parallel to the walls, which has been applied successfully and proved to be superior to other techniques and, can be used with confidence for developing turbulent flow.

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Biography

SABAH TAMIMI is currently an associate professor teaching at the College of Engineering and Computing, Al Ghurair University, Dubai,UAE. He earned his an M.Sc. in Computer Science (1988) and a Ph.D. in Applied Computer Science (1992), both of from University of Wales, UK. His interests are in computer modeling and simulation, computational fluid dynamics, software testing techniques, computer graphics and databases. During the last 11 years, he has been involved in administrative sector working as deputy dean and dean as well as faculty. He has a very good number of publications in international journals and conferences, and he is an editorial board member of several international journals.